# From predicate transformers to predicates

Dedicated by the Tuesday Afternoon Club to C.A.R. Hoare at the occasion of his being elected Fellow of the Royal Society.

**Lemma 0.** For any statement $S$ and any constant predicate $C$

$$wp(S,C) = wp(S,T) \wedge C \quad .$$

**Proof.** By substituting for $C$ the two constant predicates $T$ and $F$, respectively. (End of Proof.)

**Lemma 1.** For any statement $S$, any predicate $R$, and any constant predicate $C$

$$wp(S, R \vee C) = wp(S,R) \vee wp(S,C) \quad .$$

**Proof.** By substituting for $C$ the two constant predicates $T$ and $F$, respectively. (End of Proof.)

In the following, $P$ is a predicate in $x$ and by definition $P' = P^x_{x'}$; variables $x$ and $x'$ range over the same non-empty domain.

**Lemma 2.** For any predicate $P$ in $x$ we have for all $x$

$$P = (\underline{A}\, x' :: x \neq x' \vee P') \quad .$$

**Proof.** $P = (\underline{A}\, x' :: P) = (\underline{A}\, x' : x' = x : P') = (\underline{A}\, x' :: x \neq x' \vee P')$ .
(End of Proof.)

<u>Theorem</u>. For any statement $S$ with state space $x$ and any predicate $P$ we have

$$wp(S,P) = wp(S,T) \wedge (\underline{A}\, x' :: wp(S, x \neq x') \vee P') \quad .$$

<u>Proof</u>. $wp(S,P)$
$= \{\text{Lemma 2}\}$
$wp(S, (\underline{A}\, x' :: x \neq x' \vee P'))$
$= \{\text{distributivity of } wp \text{ over universal quantification}\}$
$(\underline{A}\, x' :: wp(S, x \neq x' \vee P'))$
$= \{\text{Lemma 1}\}$
$(\underline{A}\, x' :: wp(S, x \neq x') \vee wp(S, P'))$
$= \{\text{Lemma 0}\}$
$(\underline{A}\, x' :: wp(S, x \neq x') \vee wp(S,T) \wedge P')$
$= \{wp(S,R) \Rightarrow wp(S,T)\}$
$wp(S,T) \wedge (\underline{A}\, x' :: wp(S, x \neq x') \vee P')$ .
(End of Proof.)

Hence, the predicate transformer $wp(S,?)$ is fully characterized by the two predicates $wp(S,T)$ and $wp(S, x \neq x')$ .

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