On partitioning predicates

Let the predicates B_i satisfy

(0) $[(\underline{E}i :: B_i)]$ and

(1) $[(\underline{A}i,j : i \neq j : \neg B_i \vee \neg B_j)]$.

Such a set of predicates is called "a set of partitioning predicates".

Let functions c and d be defined by

(2) $[c R \equiv (\underline{A}i :: \neg B_i \vee R_i)]$ for any R

(3) $[d R \equiv (\underline{E}i :: B_i \wedge R_i)]$ for any R .

For partitioning predicates B_i we then have

(5) $[c R \equiv d R]$,

which is an immediate consequence of

(6) $[(\underline{E}i :: B_i)] \equiv [c R \Rightarrow d R]$, and

(7) $[(\underline{A}i,j : i \neq j : \neg B_i \vee \neg B_j)] \Rightarrow [d R \Rightarrow c R]$.

Proof of (6)

$[(\underline{E}i :: B_i)]$

$=$ {pred. calc.}

$[(\underline{E}i :: (B_i \wedge \neg R_i) \vee (B_i \wedge R_i))]$

$=$ {pred. calc.}

$[(\underline{E}i :: B_i \wedge \neg R_i) \vee (\underline{E}i :: B_i \wedge R_i)]$

$=$ {pred. calc.}

$[(\underline{A}i :: \neg Bi \vee Ri) \Rrightarrow (\underline{E}i :: Bi \wedge Ri)]$
$= \{(2) \text{ and } (3)\}$
$[cR \Rrightarrow dR]$

(End of Proof of (6).)

Proof of (7)

$[(\underline{A}i,j : i \neq j : \neg Bi \vee \neg Bj)]$
$\Rightarrow \{\text{pred. calc}\}$
$[(\underline{A}i,j : i \neq j : (\neg Bi \vee \neg Ri) \vee (\neg Bj \vee Rj))]$
$= \{\text{pred. calc.: the added conjunct is } \mathbf{true}\}$
$[(\underline{A}i,j : i \neq j : (\neg Bi \vee \neg Ri) \vee (\neg Bj \vee Rj)) \wedge$
$(\underline{A}i,j : i = j : (\neg Bi \vee \neg Ri) \vee (\neg Bj \vee Rj))]$
$= \{\text{pred. calc.}\}$
$[(\underline{A}i,j :: (\neg Bi \vee \neg Ri) \vee (\neg Bj \vee Rj))]$
$= \{\text{disjunction distributes over universal quantification}\}$
$[(\underline{A}i :: \neg Bi \vee \neg Ri) \vee (\underline{A}j :: \neg Bj \vee Rj)]$
$= \{\text{pred. calc.}\}$
$[(\underline{E}i :: Bi \wedge Ri) \Rrightarrow (\underline{A}j :: \neg Bj \vee Rj)]$
$= \{(2) \text{ and } (3)\}$
$[dR \Rrightarrow cR]$

(End of Proof of (7).)

Next we observe

$\mathbf{true}$
$= \{(5)\}$
$[\neg cR \equiv \neg dR]$
$= \{(3)\}$
$[\neg cR \equiv \neg(\underline{E}i :: Bi \wedge Ri)]$
$= \{\text{pred. calc.}\}$
$[\neg cR \equiv (\underline{A}i :: \neg Bi \vee \neg Ri)]$
$= \{(2)\}$

(8) $[\neg c R \equiv c(\neg R)]$;

$$
\begin{aligned}
& \text{true} \\
= & \{(2)\} \\
& [c(P \wedge R) \equiv (\mathbf{A}i :: \neg Bi \vee (Pi \wedge Ri))] \\
= & \{\text{pred. calc}\} \\
& [c(P \wedge R) \equiv (\mathbf{A}i :: (\neg Bi \vee Pi) \wedge (\neg Bi \vee Ri))] \\
= & \{\text{pred. calc.}\} \\
& [c(P \wedge R) \equiv (\mathbf{A}i :: \neg Bi \vee Pi) \wedge (\mathbf{A}i :: \neg Bi \vee Ri)] \\
= & \{(2)\}
\end{aligned}
$$

(9) $[c(P \wedge R) \equiv cP \wedge cR]$;

and similarly

(10) $[d(P \vee R) \equiv dP \vee dR]$.

From (5), (8), (9), and (10) we conclude that the function given by c (or d) distributes over the logical connectives (as it should). So much on partitioning predicates.

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23 July 1984

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