A theorem about "factors" perhaps worth recording

Let $\backslash$ be defined by

(0) $\qquad [x;y \Rightarrow z] \equiv [y \Rightarrow x\backslash z] \qquad ;$

(0) defines $x\backslash z$ as the weakest solution of

$$y:\ [x;y \Rightarrow z]$$

and yields with instantiations $y := x\backslash z$ and $z := x;y$ respectively

(1) $\qquad [x;\, x\backslash z \Rightarrow z]$

(2) $\qquad [y \Rightarrow x\backslash(x;y)] \qquad .$

(We have given $\backslash$ a higher binding power than $;$.)

About the transpose $\sim$(prefix) – which others call the converse $\circ$ (postfix) – I shall use the Dedekind Law

(3) $\qquad [x;y \wedge z \Rightarrow x;(y \wedge \sim x;z)] \qquad .$

We shall now prove

(4) $\qquad [p;q \wedge \sim p\backslash r \Rightarrow p;r]$

To this end we observe for any p, q, r

$\quad p;q \wedge \sim p\backslash r$

$\Rightarrow \quad \{(3) \text{ with } x,y,z := p,q,\sim p\backslash r\}$

$$
\begin{array}{ll}
 & p;(q \wedge {\sim}p;{\sim}p\backslash r) \\
\Rightarrow & \quad \{\text{monotonicities , (1) with } x,z := {\sim}p, r\} \\
 & p;r \quad .
\end{array}
$$

I used (4) to prove the theorem of section 2.3 of "A Graphical Calculus" by Sharon Curtis and Gavin Lowe, Oxford University Computing Laboratory, Parks Road, Oxford, OX1 3QD; the formulation of (4) was triggered by their note.

An alternative formulation of (4) that incorporates the antimonotonicity of $\backslash$ in its left argument is

(5) $\quad [{\sim}p \Rightarrow s] \Rightarrow [\, p;q \wedge s\backslash r \Rightarrow p;r\,]$.

I think the theorem is worth recording though not worth remembering.

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