A short sequel to EWD863

In EWD863, I left at the end the derivation of de Morgan's Laws as an exercise to the reader. The following proof, however, is too beautiful to remain unrecorded. I recall - with numbers as in EWD863 - the axioms

$$[P \wedge Q \equiv P \equiv Q \equiv P \vee Q] \qquad (9)$$

$$[P \vee \neg Q \equiv P \vee Q \equiv P] \qquad (14)$$

and the theorems

$$[\neg\neg Q \equiv Q] \qquad (19)$$

$$[\neg(P \equiv Q) \equiv \neg P \equiv Q] \quad . \qquad (20)$$

From (14), we derive with $P := \neg P$ and $P, Q := Q, P$ respectively:

$$[\neg P \vee \neg Q \equiv \neg P \vee Q \equiv \neg P]$$

$$[Q \vee \neg P \equiv Q \vee P \equiv Q] \quad ;$$

from those two with Leibniz's Principle (and symmetry of $\vee$ and $\equiv$)

$$[\neg P \vee \neg Q \equiv \neg P \equiv Q \equiv P \vee Q] \; ;$$

from that one with (20)

$$[\neg P \vee \neg Q \equiv \neg(P \equiv Q \equiv P \vee Q)] \, ,$$

and finally with (9)

$$[\neg P \vee \neg Q \equiv \neg(P \wedge Q)] \qquad (25)$$

With (19) – and $[\neg P \equiv \neg P]$, which is a syntactic descendant of $[P \equiv P]$ –

$$[\neg P \wedge \neg Q \equiv \neg(P \vee Q)] \qquad (26)$$

follows readily from (25).

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PS. The <u>single</u> substitution $P, Q := \neg Q, P$ into (14), yielding

$$[\neg Q \vee \neg P \equiv \neg Q \vee P \equiv \neg Q]$$

would have sufficed in subsequent combination with (14).

EWD.

(End of PS.)