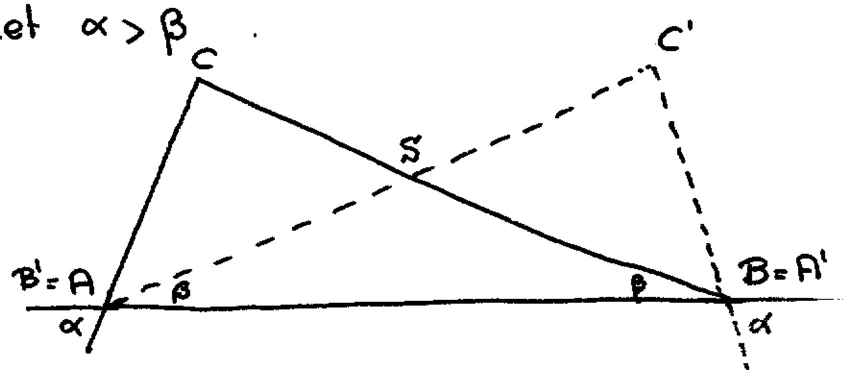
EWD974-0

The longer side is opposite to the greater angle

The introductory chapters of texts on Euclidean geometry are traditionally fishy as Euclid's axioms don't really suffice. This treatment of the theorem mentioned in the title is no better, but may have the advantage of displaying its incompleteness quite openly.

Let $\alpha > \beta$.



I need the postulate that a given geometrical figure – and its "mirror image", whatever that may be! – can exist everywhere in the plane with the same internal measures. Then we can place the mirrored triangle $A'B'C'$ with $B'=A$ and $A'=B$ and C' at the same "side" – do we know that a straight line divides the plane into two parts? – as the base. The final postulate should guarantee that, since $\alpha > \beta$, BC and $B'C'$ intersect in an interior point of these segments. Once you have swallowed all that, the rest is easy.

$$AC < B'S + SC \qquad \text{(Euclidean axiom)}$$
$$A'C' < BS + SC' \qquad \text{"}$$
$$\overline{AC + A'C' < BC + B'C'}$$

But also $AC = A'C'$ and $BC = B'C'$, hence $AC < BC$.

The latter part of the argument is nice, because it does not require that Δ ASB is isosceles.

All that has to be swallowed holds on the sphere as well, but the characterization of the straight line connecting two points as their shortest connection is on the sphere dubious; so is the theorem, so that is fine.

Is there a workable axiomatisation of the Euclidean plane, so that we could really do geometry without pictures?

So much for one more illustration of the fact that the use of pictures usually betrays that the author does not really know the rules of his game.

Strange: in my youth, Euclidean geometry was presented as axiomatised, deductive discipline, whereas algebra was presented without any axioms. The other way around seems easier (possibly with the Church-Rosser property as unproved theorem).

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