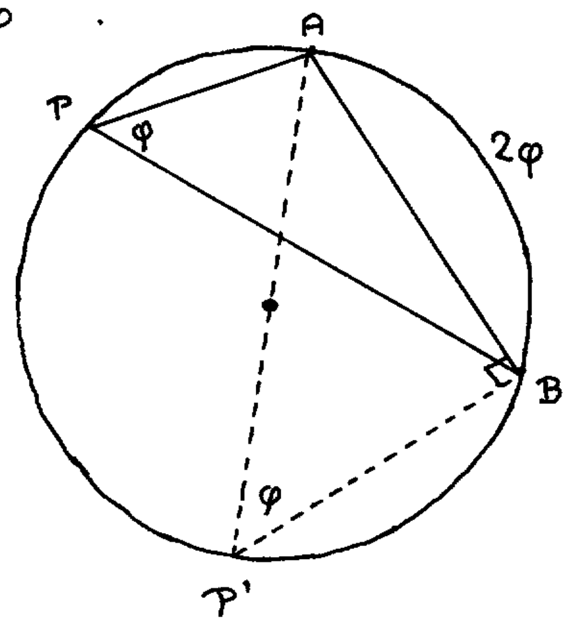
The formula for sin.$(\alpha+\beta)$

About circles we use the following

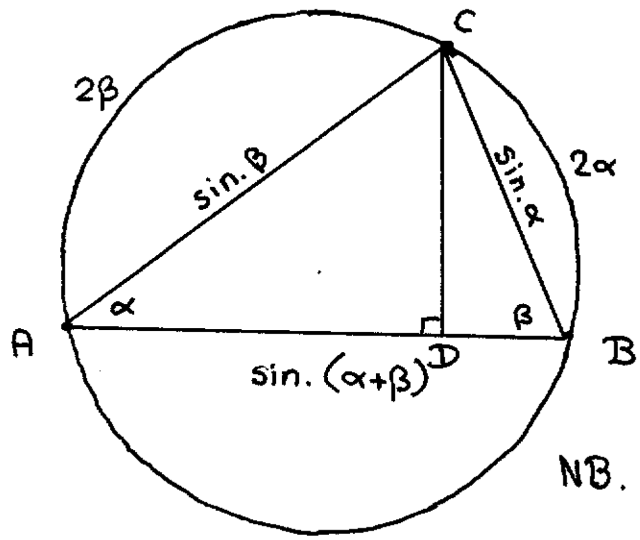
Lemma Let arc AB of a circle with diameter d subtend at the centre an angle of 2φ. Then

(i) at any point P on the remainder of the periphery it subtends an angle of φ, and

(ii) the length of chord AB equals $d \cdot \sin.\varphi$.



[By choosing $P=P'$ such that AP' is a diameter, we make $\angle ABP'$ a right angle and thus see $AB = d \cdot \sin.\varphi$.]
For the rest of this note we choose $d=1$.



NB. $2\alpha + 2\beta = 2(\alpha+\beta)$.

With the angles at A and B equal to α and β respectively, we have according to our lemma

$$BC = \sin.\alpha \qquad AC = \sin.\beta \qquad AB = \sin.(\alpha+\beta)$$

and now observe, with CD the altitude on AB

$$\begin{aligned} & \sin.(\alpha+b) \\ = \; & AB \\ = \; & AD + DB \\ = \; & AC \cdot \cos.\alpha + BC \cdot \cos.\beta \\ = \; & \sin.\beta \cdot \cos.\alpha + \sin.\alpha \cdot \cos.\beta \end{aligned}$$

which establishes the addition formula for $\sin.(\alpha+\beta)$ for $0 \leq \alpha, \beta \leq \pi/2$.

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